

An analytical study of pulsating laminar heat convection in a circular tube with constant heat flux

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Abstract

Pulsating laminar convection heat transfer in a circular tube with constant wall heat flux is investigated analytically. The results show that both the temperature profile and the Nusselt number fluctuate periodically about the solution for steady laminar convection, with the fluctuation amplitude depending on the dimensionless pulsation frequency, ω^* , the amplitude, γ , and the Prandtl number, Pr . It is also shown that pulsation has no effect on the time-average Nusselt numbers for pulsating convection heat transfer in a circular tube with constant wall heat flux.

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1. Introduction

An oscillatory flow can be classified as either a pulsating flow (with a time-mean flow) or a reciprocating flow (with a zero time-mean flow). Extensive attention has been paid to the behavior of pulsating/oscillatory flow and associated convection heat transfer for a few decades because it is frequently encountered in many practical applications. As early as in 1929, Richardson and Tyler [1] measured the velocity profile of the pulsating flow in a circular tube and found the “annular effect”. Uchida [2] and Siegel and Perlmutter [3] obtained analytical solution of the velocity profile for pulsating flow in tubes and in parallel plate channels. Recently, reciprocating flows and associated heat trans-

fer behavior in circular tubes were studied by Zhao and Cheng [4–8]. Although a number of studies on convection heat transfer in pulsating flows, including experimental investigations [9–12], analytical investigations [3,13–15] and numerical simulations [17], have been reported, the conclusions drawn from the previous studies are often inconsistent and sometimes even in contradictions. One of the key issues concerning pulsating convection heat transfer in tubes is whether a superposed flow pulsation enhances heat transfer in the original steady flow. The answer to this question in the previous studies can be classified into four different opinions: (1) flow pulsation enhances heat transfer [9,13]; (2) it deteriorates heat transfer [12,14]; (3) it has no effect on heat transfer [3,10,11]; and (4) it either enhances or deteriorates heat transfer, depending on flow parameters [15–17]. In this work, we investigate analytically pulsating convection heat transfer in a circular tube heated at a constant heat flux and show that flow pulsation neither enhances nor deteriorates heat transfer in a steady flow.

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2. Analysis

2.1. Formulation

Consider an incompressible, laminar, viscous fluid pulsating in a circular tube (with radius, r_0) heated with uniform heat flux. The pulsating flow is driven by a pressure gradient that varies sinusoidally with time as

$$\frac{\partial p}{\partial x} = \left(\frac{\partial p}{\partial x}\right)_s (1 + \gamma \cos(\omega t)), \tag{1}$$

where γ is a constant that controls the amplitude of the pressure fluctuation. For fully developed convection heat transfer, the governing equations and the boundary conditions can be written as

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\rho \partial x} + \nu \left(\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{r \partial r}\right), \tag{2}$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} = \alpha \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r}\right), \tag{3}$$

$$r = 0 : \frac{\partial u}{\partial r} = 0, \quad \frac{\partial T}{\partial r} = 0 \quad \text{and} \quad r = r_0, \quad \lambda \frac{\partial T}{\partial r} = q_w. \tag{4}$$

Introducing the following dimensionless parameters:

$$r^* = \frac{r}{r_0}, \quad \omega^* = \frac{\omega r_0^2}{\nu}, \quad t^* = \frac{\nu t}{r_0^2}, \quad u^* = \frac{u}{u_m},$$

$$\Theta = \frac{T - T_0}{q_w r_0 / \lambda}, \quad X = \frac{4x}{Re_m Pr r_0}, \quad \text{and} \quad Re_m = \frac{2u_m r_0}{\nu},$$

$$\Theta_t = -64\gamma \sum_{n=1}^{\infty} \frac{[(\lambda_n^{(1)})^2 E_{1n} + \omega^* Pr E_{2n}] \cos(\omega^* t^*) - (\lambda_n^{(1)})^2 E_{2n} - \omega^* Pr E_{1n}] \sin(\omega^* t^*)}{\omega^{*2} Pr^2 + \lambda_n^{(1)4}} \frac{J_0(\lambda_n^{(1)} r^*)}{J_0(\lambda_n^{(1)})}, \tag{13}$$

where $u_m = -\frac{r_0^2}{8\mu} \left(\frac{\partial p}{\partial x}\right)_s$ is the cross-sectional mean velocity for the time-averaged flow, we can rewrite Eqs. (2) and (3) and the corresponding boundary conditions as

$$\frac{\partial u^*}{\partial t^*} = 8[1 + \gamma \cos(\omega^* t^*)] + \left(\frac{\partial^2 u^*}{\partial r^{*2}} + \frac{\partial u^*}{r^* \partial r^*}\right), \tag{5}$$

$$Pr \frac{\partial \Theta}{\partial t^*} + 2u^* \frac{\partial \Theta}{\partial X} = \frac{\partial}{\partial r^*} \left(r^* \frac{\partial \Theta}{\partial r^*}\right), \tag{6}$$

$$r^* = 0 : \frac{\partial u^*}{\partial r^*} = 0, \quad \frac{\partial \Theta}{\partial r^*} = 0 \quad \text{and}$$

$$r^* = 1 : u^* = 0, \quad \frac{\partial \Theta}{\partial r^*} = 1. \tag{7}$$

2.2. Velocity profile

Assuming that the dimensionless velocity u^* is given by

$$u^*(r^*, t) = u_s^*(r^*) + u_t^*(r^*, t), \tag{8}$$

where $u_s^* = u_s/u_m$ is steady flow velocity, while $u_t^* = u_t/u_m$ is the imposed unsteady velocity component, and substituting Eq. (8) into Eq. (5), we obtain the velocity distribution as

$$u_s^* = 2(1 - r^{*2}), \tag{9}$$

$$u_t^* = 16\gamma \sum_{n=1}^{\infty} \frac{[\lambda_n^{(0)2} \cos(\omega^* t^*) + \omega^* \sin(\omega^* t^*)] J_0(\lambda_n^{(0)} r^*)}{\lambda_n^{(0)} J_1(\lambda_n^{(0)}) (i\omega^{*2} + \lambda_n^{(0)4})}, \tag{10}$$

where J_0 and J_1 are the Bessel function of the first kind of order 0 and 1, respectively, $\lambda_n^{(0)}$ is the eigenvalue of the Bessel function of the first kind of order 0.

2.3. Temperature profile

Similarly, assuming that the dimensionless temperature consists of a steady component and a instantaneous component [14,15], i.e.,

$$\Theta = \Theta_s(r^*, X) + \Theta_t(r^*, t) \tag{11}$$

and substituting Eqs. (8) and (11) into Eq. (6), we obtain the temperature distribution as follows:

$$\Theta_s = X + r^{*2} - \frac{r^{*4}}{4} - \frac{7}{24}, \tag{12}$$

where

$$E_{1n} = \sum_{m=1}^{\infty} \frac{\lambda_m^{(0)2}}{(\omega^{*2} + \lambda_m^{(0)4})(\lambda_m^{(0)2} - \lambda_n^{(1)2})},$$

$$E_{2n} = \sum_{m=1}^{\infty} \frac{-\omega^*}{(\omega^{*2} + \lambda_m^{(0)4})(\lambda_m^{(0)2} - \lambda_n^{(1)2})},$$

$\lambda_n^{(0)}$ and $\lambda_n^{(1)}$ are the eigenvalues of the Bessel function of the first kind of order 0 and 1.

2.4. Nusselt number

We now define the Nusselt number as

$$Nu_t = \frac{2q_w r_0}{(T_w - T_b)\lambda} = \frac{2}{\Theta_w - \Theta_{bt}}, \tag{14}$$

where Θ_{bt} is the transient bulk temperature defined as

$$\Theta_{bt} = \frac{\int_0^1 \Theta_t u_t^* dr^*}{\int_0^1 u_t^* dr^*} \tag{15}$$

Substituting the obtained velocity and temperature distribution given by Eqs. (8)–(13) into Eqs. (14) and (15), we can obtain the transient Nusselt numbers.

3. Discussion

Fig. 1 shows the transient Nusselt numbers, obtained from Eq. (14) over one period for a given pressure oscillating amplitude γ , for different Prandtl numbers Pr and dimensionless pulsation frequency, ω^* . It is seen from Fig. 1 that the Nusselt number fluctuates periodically; its fluctuation amplitude varies with the dimensionless pulsating frequency and Prandtl number. A higher pulsation frequency, ω^* , and a lower Prandtl number, Pr , lead to a smaller fluctuation amplitude of the Nusselt number. For small dimensionless pulsating frequencies, e.g., $\omega^* = 0.1$, the influence of Pr on Nu becomes negligibly small (see Fig. 1a). On the other hand, when ω^* is larger than 50, the fluctuation of Nu is smaller than 1% (see Fig. 1d). It is also interesting to observe that the time-average Nusselt number in Fig. 1(a–d) is equal to 4.36, which is the same as that for steady convection heat transfer in a circular tube with constant heat flux. This finding can be further elaborated as follows.

Following the procedures proposed by Guo [18], we now integrate Eq. (6) to obtain

$$Nu_t = Re_m Pr \int_0^1 u_s^* \cdot \nabla \Theta_s dr^* + Pr \int_0^1 \frac{\partial \Theta}{\partial t^*} dr^* \tag{16}$$

Substituting Eqs. (8) and (11) into Eq. (16) yields

$$Nu_t = Re_m Pr \left(\int_0^1 u_s^* \cdot \nabla \Theta_s dr^* + \int_0^1 u_t^* \cdot \nabla \Theta_s dr^* + \int_0^1 u_s^* \cdot \nabla \Theta_t dr^* + \int_0^1 u_t^* \cdot \nabla \Theta_t dr^* \right) + Pr \int_0^1 \frac{\partial \Theta}{\partial t^*} dr^* \tag{17}$$

Noting that the vectors u_s^* and u_t^* have the axial component only, whereas the vector $\nabla \Theta_t$ has the radial component only. It follows that their dot product is zero, i.e.

$$u_s^* \cdot \nabla \Theta_t = 0, \quad u_t^* \cdot \nabla \Theta_t = 0.$$

Hence, the third and fourth term on the right-hand side of Eq. (17) can be dropped. With this, we integrate Eq. (17) over a period T to give the time-average value Nusselt number as

$$\overline{Nu_t} = \frac{Re_m Pr}{T} \int_0^T \left(\int_0^1 u_s^* \cdot \nabla \Theta_s dr^* + \int_0^1 u_t^* \cdot \nabla \Theta_s dr^* \right) dt + Pr \int_0^T \int_0^1 \frac{\partial \Theta}{\partial t^*} dr^* dt \tag{18}$$

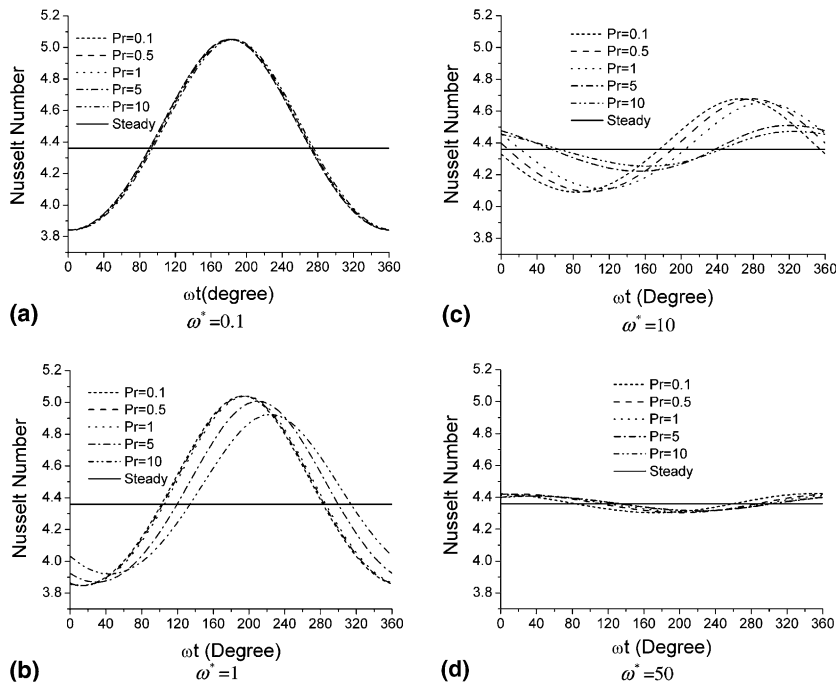


Fig. 1. Transient Nusselt numbers for different Prandtl numbers Pr and dimensionless pulsation frequency ω^* . (a) $\omega^* = 0.1$, (b) $\omega^* = 1$, (c) $\omega^* = 10$ and (d) $\omega^* = 50$.

For convenience, u_i^* in Eq. (10) can be rewritten as

$$u_i^* = f_1(r^*) \cos(\omega^* t^*) + f_2(r^*) \sin(\omega^* t^*). \quad (19)$$

Based on Eqs. (12) and (19), the three components $[r, \theta, x]$ of the vectors $\nabla\Theta_s$ and u_i^* are

$$\nabla\Theta_s : \left[\frac{2r^*}{r_0} - \frac{r^{*3}}{r_0}, 0, \frac{4}{Re_m Pr r_0} \right],$$

$$u_i^* : [0, 0, f_1(r^*) \cos(\omega^* t^*) + f_2(r^*) \sin(\omega^* t^*)]$$

and hence their dot product become

$$u_i^* \cdot \nabla\Theta_s = \frac{4f_1(r^*)}{Re_m Pr r_0} \cos(\omega^* t^*) + \frac{4f_2(r^*)}{Re_m Pr r_0} \sin(\omega^* t^*). \quad (20)$$

We now rewrite the second term on the right-hand of Eq. (18) as

$$\begin{aligned} \frac{Re_m Pr}{T} \int_0^T \int_0^1 u_i^* \cdot \nabla\Theta_s dr^* dt \\ = \frac{Re_m Pr}{T} \int_0^1 \left(\int_0^T u_i^* \cdot \nabla\Theta_s dt \right) dr^*. \end{aligned} \quad (21)$$

Substituting Eq. (20) into (21) leads us to conclude that

$$\begin{aligned} \frac{Re_m Pr}{T} \int_0^1 \left[\frac{4f_1(r^*)}{Re_m Pr r_0} \int_0^T \cos(\omega^* t^*) dt \right. \\ \left. + \frac{4f_2(r^*)}{Re_m Pr r_0} \int_0^T \sin(\omega^* t^*) dt \right] dr^* = 0. \end{aligned} \quad (22)$$

Hence, the second term on the right-hand of Eq. (18) can now be dropped. Furthermore, for the periodical pulsating flow under consideration, it is straightforward to show that the last term on the right-hand of Eq. (18) is

$$\begin{aligned} \frac{Pr}{T} \int_0^T \int_0^1 \frac{\partial\Theta}{\partial t^*} dr^* dt = \frac{Pr}{T} \int_0^1 \left(\int_0^T \frac{\partial\Theta}{\partial t^*} dt \right) dr^* \\ = \frac{Pr}{T} \int_0^1 (\Theta_T - \Theta_0) dr^* = 0. \end{aligned} \quad (23)$$

In summary, Eq. (18) finally reduces to

$$\overline{Nu_i} = Re_m Pr \int_0^1 u_s^* \cdot \nabla\Theta_s dr^*. \quad (24)$$

Eq. (24) is the same as that obtained for a steady flow by Guo [18]. The above discussion explains why the time-average Nusselt number for pulsating flow, shown in Fig. 1, is equal to the value for convection in a steady flow.

4. Concluding remarks

An analytical solution for pulsating laminar convection heat transfer in a circular tube with constant heat flux has been obtained. The result shows that an imposed flow pulsation causes both the temperature and

Nusselt number fluctuate periodically about the solution for steady laminar convection, with the fluctuation amplitude depending on the ω^* , γ and Pr . For $\gamma = 0.5$ and dimensionless frequency $\omega^* > 10$, the fluctuation in the Nusselt number is negligibly small. Finally, it has been demonstrated rigorously that that pulsation has no effect on the time-average Nusselt numbers for pulsating convection heat transfer in a circular tube with constant wall heat flux.

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References

- [1] E.G. Richardson, E. Tyler, The transverse velocity gradient near the mouths of pipes in which an alternating or continuous flow is established, Proc. Phys. Soc., London 42 (1929) 1–15.
- [2] S. Uchida, The pulsating viscous flow superposed on the steady laminar motion of incompressible fluid in a circular pipe, Zamp 7 (1956) 403–422.
- [3] R. Siegel, M. Perlmutter, Heat transfer for pulsating laminar duct flow, ASME J. Heat Transfer (May) (1962) 111–123.
- [4] T.S. Zhao, P. Cheng, A numerical solution to laminar forced convection in a heated pipe subjected to a periodically reversing flow, Int. J. Heat Mass Transfer 38 (16) (1995) 3011–3022.
- [5] T.S. Zhao, P. Cheng, Oscillatory heat transfer in a pipe subjected a periodically reversing flow, ASME J. Heat Transfer 118 (1996) 592–598.
- [6] T.S. Zhao, P. Cheng, The friction coefficient of laminar oscillatory flow in a circular pipe, Int. J. Heat Fluid Flow 17 (1996) 167–172.
- [7] T.S. Zhao, P. Cheng, Experimental investigations of the onset of turbulence and frictional losses in an oscillatory pipe flow, Int. J. Heat Fluid Flow 17 (1996) 356–362.
- [8] T.S. Zhao, P. Cheng, Oscillatory pressure drops through a woven-screen packed column subjected to a cyclic flow, Cryogenics 36 (1996) 333–341.
- [9] X. Shuai, S. Cheng, G. Antonini, Amelioration du transfert convectif de chaleur par l'doulement pulse dans un fluide visqueux, The Can. J. Chem. Eng. 72 (June) (1994) 468–475.
- [10] M.R. Mackley, G.M. Tweddle, I.D. Wyatt, Experimental heat transfer measurements for pulsatile flow in baffled tubes, Chem. Sci. Eng. 45 (5) (1990) 1237–1242.
- [11] M.R. Mackley, P. Stonestreet, Heat transfer and associated energy dissipation for oscillatory flow in baffled tubes, Chem. Sci. Eng. 50 (14) (1995) 2211–2224.
- [12] V.V. Mamayev, V.S. Nosov, N.I. Syromyatnikov, Investigation of heat transfer in pulsed flow of air in pipes, Heat Transfer—Sov. Res. 8 (3) (1976) 111–116.

- [13] M. Faghri, K. Javadani, A. Faghri, Heat transfer with laminar pulsating flow in a pipe, *Lett. Heat Mass Transfer* 6 (1979) 259–270.
- [14] H.N. Hemida, M.N. Sabry, A. Abdel-Rahim, H. Mansour, Theoretical analysis of heat transfer in laminar pulsating flow, *Int. J. Heat Mass Transfer* 45 (2002) 1767–1780.
- [15] T. Moschandreou, M. Zamir, Heat transfer in a tube with pulsating flow and constant heat flux, *Int. J. Heat Mass Transfer* 40 (10) (1997) 2461–2466.
- [16] S.Y. Kim, B.H. Kang, J.M. Hyun, Heat transfer in the thermally developing region of a pulsating channel flow, *Int. J. Heat Mass Transfer* 36 (17) (1993) 4257–4266.
- [17] H.W. Cho, J.M. Hyun, Numerical solution of pulsating flow and heat transfer characteristics in a pipe, *Int. J. Heat Fluid Flow* 11 (4) (1990) 321–330.
- [18] Z.Y. Guo, Mechanism and control of convective heat transfer—Coordination of velocity and heat flow fields, *Chin. Sci. Bull.* 46 (7) (2001) 596–599.